RECTANGLE INSIDE

SZYMON ŻEBERSKI

Two-dimensional version of the classical Mycielski theorem says that for every comeager or conull set $X \subseteq [0,1]^2$ there exists a perfect set $P \subseteq [0,1]$ such that $P \times P \subseteq X \cup \Delta$. We consider strengthening of this theorem by replacing a perfect square with a rectangle $A \times B$, where A and B are bodies of some types of trees with $A \subseteq B$. In particular, we show that for every comeager G_δ set $G \subseteq \omega^\omega \times \omega^\omega$ there exist a Miller tree T_M and a uniformly perfect tree $T_P \subseteq T_M$ such that $[T_P] \times [T_M] \subseteq G \cup \Delta$ and that T_P cannot be a Miller tree. In the case of measure we show that for every subset F of $2^\omega \times 2^\omega$ of full measure there exists a uniformly perfect tree $T_P \subseteq 2^{<\omega}$ such that $[T_P] \times [T_P] \subseteq F \cup \Delta$ and no side of such a rectangle can be a body of a Silver tree or a Miller tree. We also show some properties of forcing extensions from which we derive nonstandard proofs of Mycielski-like theorems via Shoenfield Absoluteness Theorem.

Presented results are obtained together with M. Michalski and R. Rałowski.

References

[1] H. G. Eggleston, Two measure properties of Cartesian product sets, The Quarterly Journal of Mathematics 5 (1954), 108–115,

[2] M. Michalski, R. Rałowski, Sz. Żeberski, Mycielski among trees, Mathematical Logic Quarterly, vol. 67 (2021), 271–281,

[3] J. Mycielski, Algebraic independence and measure, Fundamenta Mathematicae 61 (1967), 165-169,

[4] Sz. Żeberski, Nonstandard proofs of Eggleston like theorems, Proceedings of the Ninth Topological Symposium (2001), 353–357.

Faculty of Pure and Applied Mathematics, Wrocław University of Science and Technogy

Email address: szymon.zeberski@pwr.edu.pl